

REQUIREMENTS FOR STIFFNESS VARIATION BETWEEN SLAB AND BALLASTED RAILWAY TRACK AND RESULTING DESIGN LOADS

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ABSTRACT

In this paper, a parametric investigation of the static and dynamic elasticity (stiffness) of the railway track and of the rail pads is presented, and the results are compared to regulation requirements (Anforderungskatalog, 2002). Moreover, the influence of the variation of the static and dynamic stiffness on the design loads induced by the stiffness variations is investigated. This is a factor of decisive importance for the dimensioning of the constitutive elements of the Slab Track, the Transition Zone and the Ballasted Track. Conclusions are drawn for the requirements of stiffness variation and the magnitude of the design loads.

KEYWORDS: Railway track, superstructure, resilient fastening, actions, stiffness, dimensioning.

INTRODUCTION

The term "Slab Track" (*Feste Fahrbahn* in German, *Voie sur Dalles* in French) defines the multilayered structure of a Railway Track which secures the seating of the track panel through a rigid reinforced concrete plate (slab), which seats on a series of successive bearing layers with a gradually decreasing modulus of elasticity, instead of a ballast-bed as in the classic ballasted track. After many years of international experience (e.g. Japan, Germany, France, etc.) in High-Speed lines, a significant damage to ballast was observed, which was literally crashed, fouled and completely compacted due to excessive dynamic loading, breaking forces, etc., resulting in loss of its resilience, the deterioration of the rain water drainage, the incapability of maintaining the geometry of the track, etc. Under these circumstances, maintenance of the track geometry within the regulations limits demands repeated and costly interventions. Moreover, the individual structural elements of superstructure (rails, sleepers, fastenings, etc.) undergo non-permissible wear and it is obligatory to be replaced in a much shorter time than their life-cycles. Furthermore, very costly interventions cannot be avoided even in substructure (Tsoukantas, 1999). All these reasons resulted in the adoption of the Slab Track technology.

The adoption of the Slab Track technology in a railway network creates the necessity to introduce Transition Zones between the Slab (Ballastless) Track and the Ballasted Track sections as interfaces. In the Transition Zones the total stiffness (elasticity) coefficient of the multilayered structure should change gradually in order to secure a smooth transition to the fluctuation of the acting forces on the track. The acting forces are a decisive factor for the dimensioning of the permanent way both for ballasted and ballastless track. This paper presents an investigation of the influence of the change of track stiffness coefficient on the acting forces and consequently on the dimensioning of the superstructure of the track (Slab Track, Transition Zone, Ballasted Track). This is performed on the occasion of the use of Rheda 2000 type Slab Track in the High-speed network ($V > 200$ km/h) of OSE, the Greek Railways (Giannakos, 2008).

1. CALCULATION METHODS OF THE DESIGN LOAD OF A RAILWAY TRACK

Probabilistic approach, adopted for the calculation of the Design Load, generally consists of the estimation of the increase of the mean value of the vertical wheel load in order to cover the statistically desirable safety level. In this frame three basic calculation methods are distinguished, characterizing, respectively, three different ways of approaching the matter:

- The method proposed in the French Bibliography (Alias, 1984; Prud'homme *et al.*, 1976; RGCF, 1973);
- The method proposed in the German Bibliography (Fastenrath, 1981; Eisenmann, 2004);
- The method proposed by Giannakos (2004, 2009).

(a) The equation cited in French bibliography (Prud'homme *et al.*, 1976) is:

$$R_{total} = (Q_{wheel} + Q_{\alpha} + 2 \cdot \sqrt{[\sigma^2(\Delta Q_{NSM})] + [\sigma^2(\Delta Q_{SM})]}) \cdot \bar{A}_{stat} \cdot 1,35 \quad (1)$$

where: Q_{wheel} = the static load of the wheel (half axle load)

Q_{α} = load due to cant (superelevation) deficiency

$\sigma(\Delta Q_{NSM})$ = standard deviation of the Non-Suspended Masses of vehicle

$\sigma(\Delta Q_{SM})$ = standard deviation of the Suspended Masses of vehicle

\bar{A}_{stat} = reaction coefficient of the sleeper which is equal to:

$$\bar{A}_{stat} = \frac{1}{2\sqrt{2}} \sqrt[4]{\frac{\rho \cdot \ell^3}{E \cdot J}} \quad (2)$$

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where ρ = coefficient of total static stiffness (elasticity) of track

ℓ = distance among the sleepers

E, J = Modulus of Elasticity and Moment of Inertia of the rail

Equation (1) gives the most adverse results among the equations cited in the French bibliography for dimensioning of the constitutive elements of track's superstructure and substructure (Prud'homme *et al.*, 1976). In practice equation (1) gives 10% higher value for reaction R than correspondent equations cited in the French bibliography (Alias, 1984; RGCF, 1973) for the most adverse conditions of track stiffness (rigid undeflected structure) for $k = 12$ which is the most adverse coefficient of the rail running table of rail (Giannakos, 2004).

(b) The equation cited in German bibliography (Fastenrath, 1981; Eisenmann, 2004) is:

$$R = S = \frac{Q_{total} \cdot \ell}{2 \cdot L} \Rightarrow R = \frac{Q_{total}}{2} \cdot \sqrt{\frac{\rho \cdot \ell^4}{4 \cdot E \cdot J \cdot \ell}} = Q_{total} \cdot \frac{1}{2\sqrt{2}} \cdot \sqrt[4]{\frac{\rho \cdot \ell^3}{E \cdot J}} = \bar{A}_{stat} \cdot Q_{total} \quad (3)$$

$$\text{where: } Q_{total} = Q_{wheel} \cdot (1 + t \cdot \bar{s}) \quad (4)$$

and Q_{wheel} = the static load of the wheel,

$\bar{s} = 0.1 \cdot \varphi$ to $0.3 \cdot \varphi$, depending on the condition of the track, that is

$\bar{s} = 0.1 \varphi$ for excellent track condition

$\bar{s} = 0.2 \varphi$ for good track condition

$\bar{s} = 0.3 \varphi$ for poor track condition

and φ is determined by the following formulas as a function of speed:

For $V < 60$ km/h, $\varphi = 1$.

For $60 < V < 200$ km/h, $\varphi = 1 + \frac{V-60}{140}$

where V is the maximum speed on a section of track and t is the coefficient dependent on the probabilistic certainty P ($t = 1$ for $P = 68.3\%$, $t = 2$ for $P = 95.5\%$ and $t = 3$ for $P = 99.7\%$).

(c) The equation that was proposed as a result of the research in the Greek railway network (Giannakos 2004, 2009) is:

$$R_{service} = \bar{A}_{dynam} \cdot (Q_{wheel} + Q_{\alpha}) + (3 \cdot \sqrt{[\sigma^2(\Delta Q_{NSM})]^2 + [\sigma^2(\Delta Q_{SM})]^2}) \quad (5)$$

$$\text{where } \bar{A}_{\text{dynam}} = \frac{1}{2\sqrt{2}} \cdot \sqrt[4]{\frac{\ell^3 \cdot h_{TR}}{E \cdot J}} \text{ and } h_{TR} = \rho_{\text{dynam}} = 2\sqrt{2} \cdot \sqrt[4]{E \cdot J \cdot \left(\frac{\rho}{\ell}\right)^3} \quad (6)$$

and ρ is the total static stiffness coefficient of the track.

2. PRECONDITIONS FOR TRANSITION ZONE, BALLASTED AND SLAB TRACK

The above equations have been applied in the cases of ballastless track, transition zone and ballasted track. For the determination of the spring constant (stiffness) of the slab track, Table 1 is valid for Ballasted and Ballastless Tracks as derived from measurements in the German railway network (Leykauf *et al.*, 1990). For Slab Track the classic Rheda type slab track was used.

	Bearing Capacity of Subgrade					
	Ballasted Track			Ballastless Track		
	poor	good	very good	Concrete slab		
Ballast Coefficient C [N/mm ³]	0.05	0.10	0.15	0.30	0.35	0.40
Sleeper Reaction coefficient p [kN/mm]	14	29	43	86	100	114

Table 1. Relation between ballast coefficient C and stiffness coefficient p (or c) in a line equipped with rails UIC60 and monoblock sleepers (ties) of prestressed concrete B70 and concrete plate/slab (Leykauf *et al.*, 1990)

The seating surface of the sleeper is $F = 5700 \text{ cm}^2$ and the distance between two consecutive sleepers is 60 cm. Bearing in mind that $\rho = C \cdot F/2$, the value of ρ for ballasted track calculated for the cases of Table 1 is:

$$\rho = C \cdot \frac{F}{2} = 0.05 \frac{1}{1000} \text{ kN/mm}^3 \cdot \frac{5700 \cdot 100 \text{ mm}^2}{2} = 14.2514 \text{ kN/mm} \quad (7a)$$

$$\rho = C \cdot \frac{F}{2} = 0.10 \frac{1}{1000} \text{ kN/mm}^3 \cdot \frac{5700 \cdot 100 \text{ mm}^2}{2} = 28.5529 \text{ kN/mm} \quad (7b)$$

$$\rho = C \cdot \frac{F}{2} = 0.15 \frac{1}{1000} \text{ kN/mm}^3 \cdot \frac{5700 \cdot 100 \text{ mm}^2}{2} = 42.7543 \text{ kN/mm} \quad (7c)$$

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The value of ρ for ballastless track calculated for the cases of Table 1 is:

$$\rho = C \cdot \frac{F}{2} = 0.30 \frac{1}{1000} \text{ kN/mm}^3 \cdot \frac{5700 \cdot 100 \text{ mm}^2}{2} = 85.586 \text{ kN/mm} \quad (8a)$$

$$\rho = C \cdot \frac{F}{2} = 0.35 \frac{1}{1000} \text{ kN/mm}^3 \cdot \frac{5700 \cdot 100 \text{ mm}^2}{2} = 99.75 \approx 100 \text{ kN/mm} \quad (8b)$$

$$\rho = C \cdot \frac{F}{2} = 0.40 \frac{1}{1000} \text{ kN/mm}^3 \cdot \frac{5700 \cdot 100 \text{ mm}^2}{2} = 114.0 = 114 \text{ kN/mm} \quad (8c)$$

In a Rheda type Slab Track the sleepers used are types of B70 with seating surface of $F = 5700 \text{ cm}^2$. Consequently, for the concrete plate, functioning as subgrade underneath the seating surface of the monoblock sleepers (B70), it will also be valid: $\rho = C \cdot F/2$. This implies that the coefficients of spring constant (stiffness coefficient) for the Slab Track can be calculated in a similar way from the aforementioned equations. In this manner the Slab Track stiffness can be calculated using the same parameters as for the case of "ballast and frozen soil" as cited in Giannakos (2004, 2009). The methodology described above simulates - for the analysis - the concrete slab (Betonplatte) as well as the layers under this slab. We should cite here Professor J. Eisenmann (1994), who references that in the New-Constructed Railway Superstructures (NBS - Neubaustrecke) in Germany the Ballast Coefficient C may reach the value of 0.60 N/mm^3 (this implies $\rho = 171 \text{ kN/mm}$), which has been measured on site and for this reason it has also been taken into account in the parametrical solution / investigation that follows.

We can verify the method presented in Giannakos (2004) using Eisenmann (1979), where it is cited that the mean value of concrete slab subsidence is 0.23 mm (fluctuating between 0.17 and 0.31 mm). This is a result almost identical to the results of the methodology used for this paper (Giannakos *et al.*, 2009). Consequently the coefficient of total static elasticity (stiffness) of track ρ_{total} for Slab Track (with concrete sleepers embedded in its structure) is given by the equation:

$$\frac{1}{\rho_{total}} = \frac{1}{\rho_{rail}} + \frac{1}{\rho_{pad1}} + \frac{1}{\underset{\text{if-it-exists}}{\rho_{pad2}}} + \frac{1}{\rho_{sleeper}} + \frac{1}{\rho_{concrete-slab}} \quad (9)$$

It must be underlined that the track is a multilayered structure of v layers (Figure 1) simulated by a combination of springs (with coefficient ρ_i [kN/mm]) and dampers (with coefficients c_i). For the total track structure the following equation applies:

$$\frac{1}{\rho_{total}} = \sum_{i=1}^v \frac{1}{\rho_i} \quad (10)$$

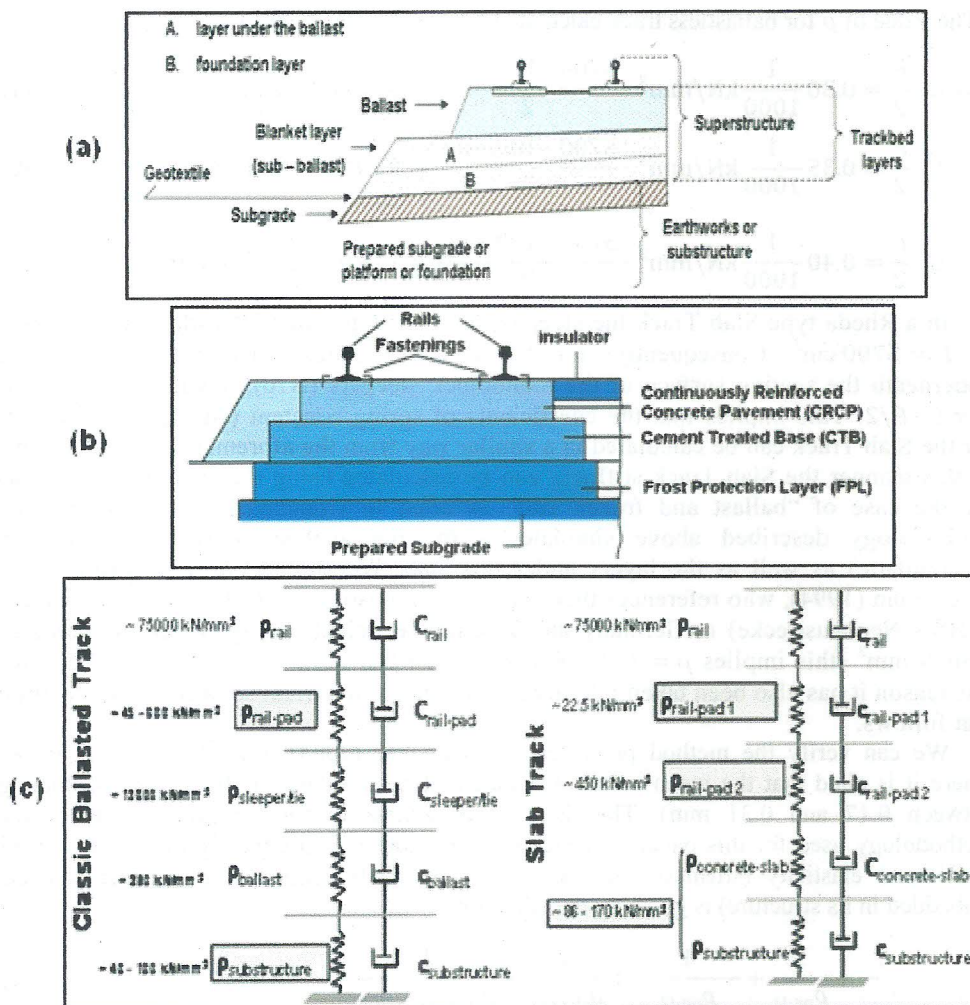


Figure 1. Track is a multilayered structure: (a) Classic Ballasted Track cross-section, (b) Slab Track cross section, (c) simulation of the track as a combination of springs with coefficient ρ_i and dampers with coefficient c_i

Here ρ_i is the coefficient of "Rail Support Modulus" (k in American bibliography (AREMA, 16-10-10)) of each layer. This implies that ρ_{total} is a coefficient of quasi elasticity (stiffness) of the track, which is the "spring constant" of Hooke's law. It is defined as the "reaction coefficient of the tie", and ρ_i is the "spring constant" of each layer. In the aforementioned

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simulation, all the layers underneath the ballast-bed in the case of ballasted track (blanket-layer, subgrade, prepared subgrade, soil) appear –in total- with the coefficient $\rho_{substructure}$. In the case of slab track all the layers beneath the rail-pad of the fastening (the concrete slab CRCP included) appear –in total- with the coefficient $\rho_{substructure}$.

The aforementioned methods were programmed in a computer code and parametric investigations were performed varying the stiffness of the substructure using the equations (7) to (9). Its results are cited in the next paragraphs.

3. INFLUENCE OF RAIL PAD STIFFNESS VARIATION ON TOTAL TRACK STIFFNESS

3.1. STATIC AND DYNAMIC COEFFICIENTS OF STIFFNESS OF THE ELASTIC PADS

For the parametric investigation the W14 Fastenings with Zw700 Wirtwein pad for ballasted track and Ioarv300 with Zw104/22.5 for Slab Track, both of Vossloh GmbH are used, which are laid in the Hellenic Railway network and are among the most resilient fastenings all over the world. The Load – Deflection curves of these fastenings were used to determine the coefficient ρ (or c) for the pads (and also the fastenings).

The investigation yielded results are depicted in Figure 2. In the upper illustration the static stiffness coefficients of the pads are presented for a range of the stiffness coefficients of the substructure (Slab Track) between 86 kN/mm and 250 kN/mm. The lower illustration depicts the dynamic stiffness coefficients of the pads (Giannakos, 2004; Giannakos *et al.*, 2009) or coefficients of track stiffness c_G according to the “List of Requirements for Slab Track Construction” of German Railways (Anforderungskatalog, 2002, 2. seite 1) for the cases of Slab Track, Transition Zone, and Ballasted Track. From Figure 2 (lower illustration) it is derived that for the Slab Track section of the permanent way (pad Zw104/22.5), the coefficient of total stiffness of track (gleisstetigkeit) c_G , as it is defined in the “List of Requirements for Slab track Construction” of German Railways (Anforderungskatalog, 2002, 2. seite 1), covers the demands of this List:

$$\rho_{dynamic-pad} = c_G = 64 \pm 5 \text{ kN/mm}$$

For the ballasted track in the “List of Requirements for Slab Track Construction” (Anforderungskatalog, 2002, Anhang 2.1 seite 3 – 4), an example is cited for the calculation of the stiffness coefficient c_G (gleisstetigkeit), where instead of ρ (or c), the sum of the inverse ρ_i of the substructure and of the pad (for $C = 0.15 \text{ N/mm}^3$, $\rho_{substructure} = 43 \text{ kN/mm}$ as in Table 1 of the present paper) is taken into account, obviously in order to facilitate calculation. In fact, the results are slightly more adverse and consequently to the safer side in comparison to the use of ρ_{total} from the equation (9) which would be more accurate but a bit more complicated.

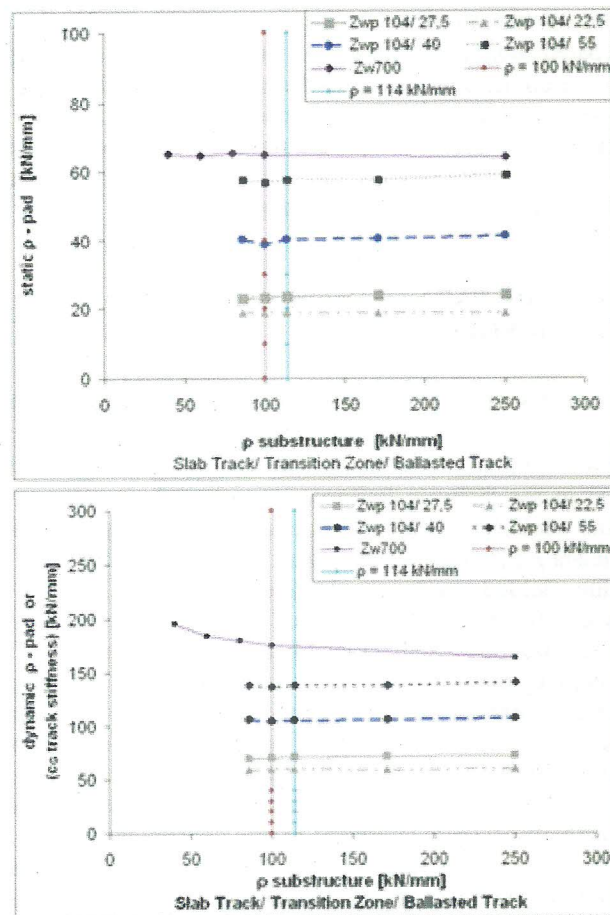


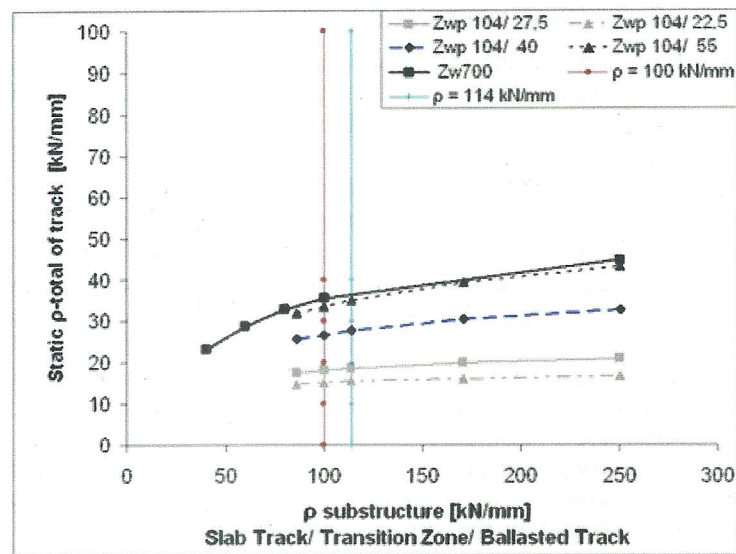
Figure 2. Coefficient of pad stiffness ρ (upper illustration) static and (lower illustration) dynamic

In the present paper the calculations are performed with the use of the more accurate ρ_{total} , that is of the coefficient of the total static stiffness of track. In this case for comparability reasons the results for $\rho_{substructure} = 40 \text{ kN/mm} \cong 43 \text{ kN/mm}$ are used (Table 1). Consequently it is derived:

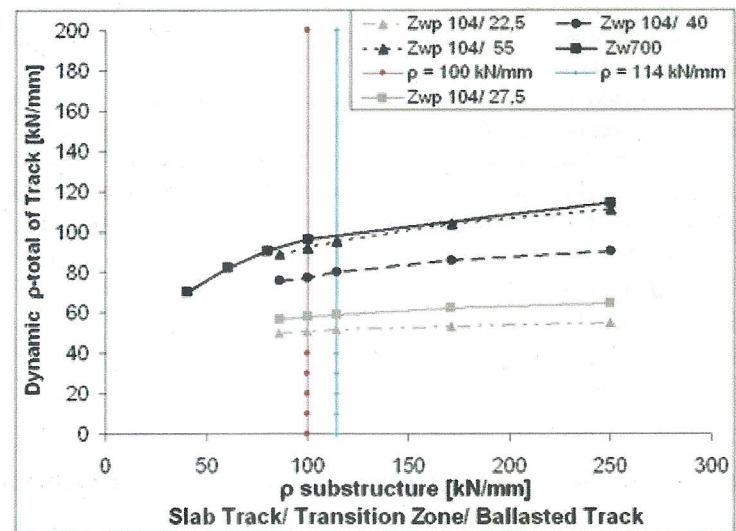
$$\rho_{total-dynamic} = h_{TR} = 67.76 \text{ kN/mm} < 78 \text{ kN/mm}$$

The result above is similar to the example of the "List of Requirements for Slab Track Construction" (Anforderungskatalog, 2002, Anhang 2.1 seite 3–4).

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(a)



(b)

Figure 3. Coefficient of total track stiffness p (a) static and (b) dynamic

3.2. STATIC AND DYNAMIC COEFFICIENT OF THE TOTAL TRACK STIFFNESS

Parametric investigation using the Load-Deflection curves of the elastic pads and ranges of stiffness coefficients as above yielded results for the coefficients of total static stiffness $\rho_{total-stat}$ of track as well as the coefficients of total dynamic stiffness of track $\rho_{total-dynamic} = h_{TR}$ (Giannakos, 2004, 2007). These results are depicted in Figure 3. In Figure 3(a) the coefficient of the total static stiffness of track $\rho_{total-stat}$ in Slab Track, Transition Zone and Ballasted Track is presented and in Figure 3(b) the coefficients of the total dynamic stiffness of track $\rho_{total-dynamic} = h_{TR}$ in Slab Track, Transition Zone and Ballasted Track. Parametric investigation shows that Slab Track presents coefficient of total dynamic stiffness of track approximately 50 % smaller than the Ballasted Track. It must be noted that even though the Slab Track is much more rigid (stiff) than the Ballasted Track due to the bearing concrete slab, after the appearance and the use of the highly resilient fastenings of advanced technology with the corresponding compatible elastic pads its overall response becomes much softer.

Moreover, it is observed that there is no significant amplitude of fluctuation of the total track stiffness coefficient for relevant subgrade stiffness fluctuation from very "soft"/flexible of 40 kN/mm in the case of gravelly subgrade, to very rigid of 250 kN/mm in the case of rocky tunnel bottom in the case of Ballasted Track, and from 84 kN/mm to 250 kN/mm in the case of Slab Track (see also Giannakos *et al.*, 2009). In Figures 2 and 3 the vertical curves for (a) $\rho = 100$ kN/mm and (b) $\rho = 114$ kN/mm are depicted which represent the stiffness coefficient of substructure for ballasted track/slab track ($\rho = 100$) and slab track ($\rho = 114$).

For New Constructed Lines NBS (Neubaustrecke) in Germany these values are the most representative according to the existing German bibliography. Parametric investigation – using the Load-Deflection curves of the elastic pads – and for fluctuation of the stiffness coefficients of the Slab Track (and the Transition Zone) from 86 kN/mm to 250 kN/mm gave results for the coefficients of total static stiffness $\rho_{total-stat}$ of track as well as the coefficients of total dynamic stiffness of track $\rho_{total-dynamic} = h_{TR}$. For fluctuation of the static coefficients of substructure for ballasted track from 40 kN/mm to 250 kN/mm (Giannakos, 2004, 2007) the results for the ρ_{stat} were calculated, for the case of ballasted track and for $\rho_{dynamic} = h_{TR}$, using the Load-Deflection curves of the relevant elastic pads of fastenings.

4. ACTIONS ON THE TRACK PANEL RESULTING FROM STIFFNESS VARIATION

In the present paper the calculations have been performed, for confidence interval (possibility of appearance) of 99.7% (Giannakos method and method of German bibliography) to 95.5 % (method of French bibliography), according to the three methodologies described above.

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It was found that the Actions (Loads) on the track superstructure in the case of Slab Track have negligible fluctuations around the level of 150 kN for subgrade stiffness varying from 84 kN/mm to 250 kN/mm (in the case of a tunnel's rocky bottom) for the Slab Track case. This should be compared to the actions of about 170 kN in the case of the Ballasted Track with fastening W14 and subgrade stiffness from very flexible 40 kN/mm of gravelly subgrade to 250 kN/mm.

The results are depicted in the Figures 4, 5, 6 and 7 and a clear comparison among the results derived by the three aforementioned methods can be done in all these 4 figures. In Figure 4 the actions on the track superstructure in the case of Slab Track are depicted, with fastening Ioarv300 of Vossloh Gmbh and elastic pad Zw104/22.5 kN/mm. In Figure 5 the actions on the track superstructure in the case of the Transition Zone are depicted, in Figure 5(a) with fastening Ioarv300 of Vossloh Gmbh and elastic pad Zw104/27.5 kN/mm and in Figure 5(b) with fastening Ioarv300 of Vossloh Gmbh and elastic pad Zw104/40 kN/mm.

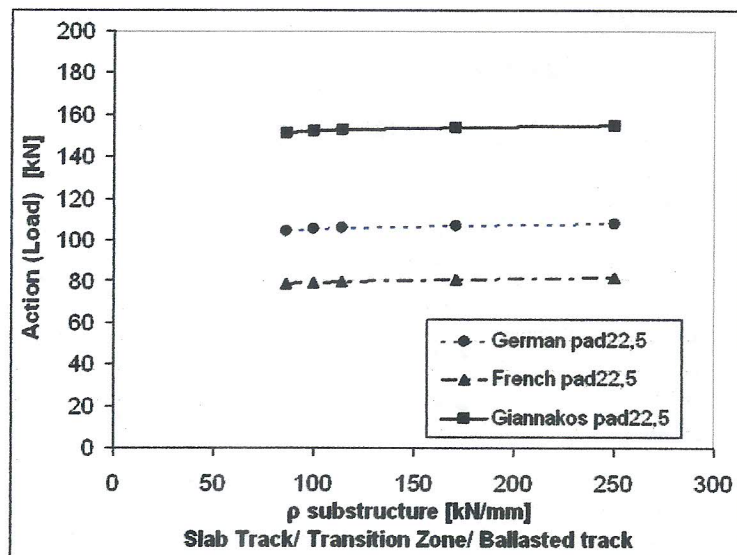
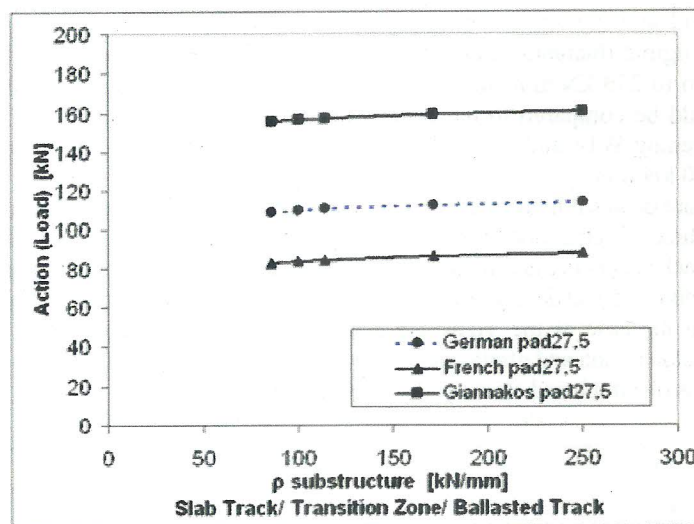
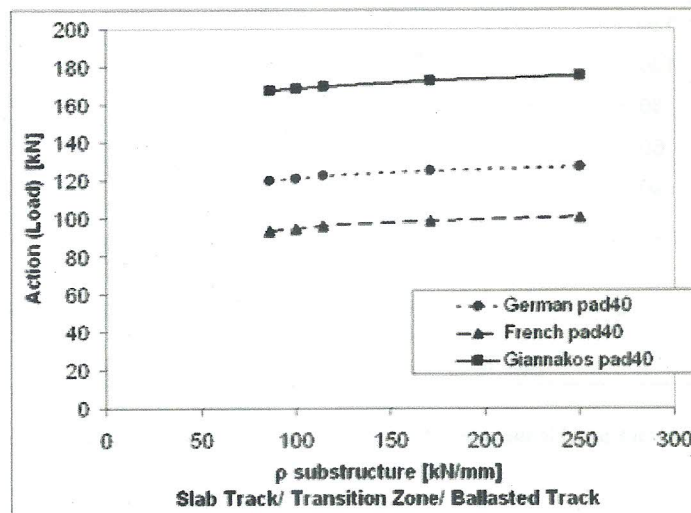


Figure 4. Actions on track panel in the case of Ioarv 300 Fastening and pad Zw104/22.5 kN/mm (Slab Track)

In Figure 6 the actions on the track superstructure in the case of the Transition Zone also are depicted, with fastening Ioarv300 of Vossloh Gmbh and elastic pad Zw104/55 kN/mm. In Figure 7 the actions on the track superstructure in the case of the Ballasted Track are depicted, with fastening W14 of Vossloh Gmbh and elastic pad Zw700 (Wirthwein).



(a)



(b)

Figure 5. Actions on track panel in the case of Ioarv 300 Fastening and (a) pad Zw104/27.5 kN/mm and (b) pad Zw104/40 kN/mm in the Transition Zone

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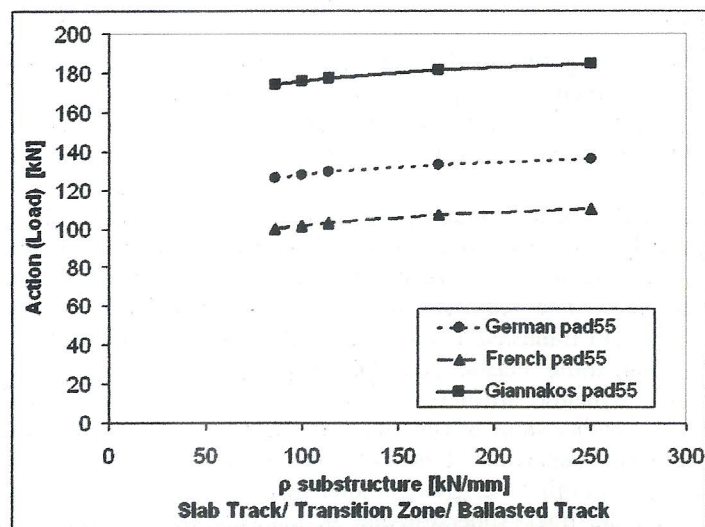


Figure 6. Actions on track panel in the case of Ioarv 300 Fastening and pad Zw104/55 kN/mm in the Transition Zone

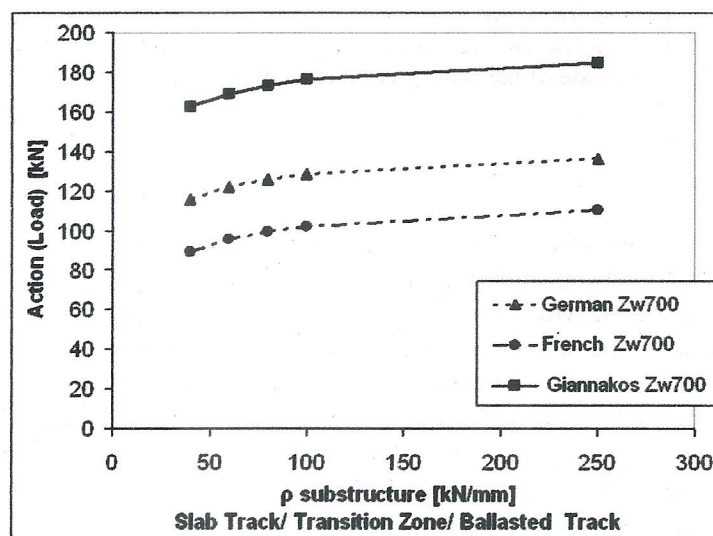


Figure 7. Actions on track panel in the case of W14 fastening and pad Zw700 in the Ballasted Track section

CONCLUSIONS

A parametric investigation of the static and dynamic stiffness of a ballasted and a Slab Track was performed. The applied loads and their impact on the dimensioning of the Slab Track, the Transition Zone and the Ballasted Track were evaluated. Results were presented for the requirements of stiffness variation and the magnitude of the design loads.

Even though Slab Track is much more rigid than the Ballasted Track due to the bearing concrete slab, it presents lower stiffness than the Ballasted Track due to the use of the highly resilient fastenings of advanced technology, with the relevant compatible to them elastic rail pads. The requirement for "smooth" elasticity (stiffness) transition between Slab Track and Ballasted Track is fulfilled by adopting high resilient fastenings combined with compatible elastic pads of changing stiffness. Slab Track presents coefficient of total dynamic stiffness of track approximately 50 % smaller than the Ballasted Track. The higher value of concrete slab stiffness compared to the ballasted track – implying more "rigid" behaviour of the structure – is counterbalanced by the much more resilient fastening pad in this case, giving a total stiffness less than in ballasted track. The Actions (Loads) on the track superstructure in the case of Slab Track with Ioarv300 fastening remain almost unchangeable to the level of 150 kN, in comparison to the actions of ≈ 170 kN in the case of Ballasted Track with fastening W14 in a wide range of subgrade stiffness. A smooth change of elasticity and consequently of the acting loads is required in order to secure the smooth rolling of the vehicles. These actions should be undertaken from the structure and thus the engineers should take them into account for the dimensioning of the concrete slab in the case of Slab Track and the dimensioning of the concrete sleepers in the case of the Ballasted Track and the Transition Zone.

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